

An iterative process for a hybrid pair of a Bregman strongly nonexpansive single-valued mapping and a finite family of Bregman relative nonexpansive multi-valued mappings in Banach spaces

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ABSTRACT.

In this paper, we construct an iterative process involving a hybrid pair of a Bregman strongly nonexpansive single-valued mapping and a finite family of Bregman relative nonexpansive multi-valued mappings and prove strong convergence theorems of the proposed iterative process in reflexive Banach spaces under appropriate conditions. Our main results can be viewed as an improvement and extension of the several results in the literature.

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